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SHORT PAPER

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Three-dimensional vortex visualization in stratified spin-up

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Abstract We present the results of three-dimensional time-dependent numerical simulations of incremental spin-up of a thermally stratified fluid. The fluid inside a vertical cylindrical container of radius R and height $2H$ is water characterized by the kinematic viscosity ν and thermal diffusivity κ. Initially, its density (temperature) varies linearly with height and is characterized by a constant buoyancy frequency N, which is proportional to the density gradient. The system undergoes an abrupt change in the rotation rate from its initial value Ω_i, when the fluid is in a solid-body rotation state, to the final value Ω_f. The aim of this contribution is to show the formation of columnar vortices in a high Rossby number spin-up flow.

Keywords Spin-up · Rotation · Stratification

1 Introduction

In this communication, we show three-dimensional (3D) vortex structures that appear as a result of baroclinic instability experienced by a thermally stratified fluid contained inside a vertical cylindrical reservoir of radius $R = 20$ cm and height $2H = 6$ cm at a high Rossby number. The baroclinic instability develops at late stages of flow development (several tens of rotation period) due to an abrupt change of the rotation rate of the cylinder (so-called spin-up) from Ω_i (0.105 rads/s), when the fluid is in a solid-body rotation, to Ω_f (0.384 rad/s). The vortex structure was recovered in the course of high-resolution 3D time-dependent numerical simulations of a spin-up process. The fluid contained inside the cylinder is water with kinematic viscosity ν and thermal diffusivity κ. Initially its temperature varies linearly with height and is characterized by a constant buoyancy frequency N (0.97 rad/s), which is proportional to the density gradient. The

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characteristic Ekman number $E = \nu/2\Omega_f H^2$ ($\sim 10^{-3}$) is small, but at the same time it guarantees that the Ekman bottom boundary layer is in the transitional state. The latter condition is satisfied when the Reynolds number, $55 < Re_\delta$ $(U\delta/\nu = 90) < 150$, where $U = (\Omega_f - \Omega_i)$, R is the characteristic velocity and $\delta = (\nu/\Omega_f)^{1/2}$ is the bottom Ekman layer depth. The forcing is characterized by the Rossby number $\varepsilon = (\Omega_f - \Omega_i)/\Omega_f$ (0.727), so that the flow regime under investigation is highly nonlinear. The importance of the temperature stratification is characterized by the Burger number $Bu = NH/\Omega_f R$ (0.38). The Prandtl number ν/κ (~ 7) is that of water at normal atmospheric conditions. The boundary conditions for the simulations were no-slip on both, the bottom and side walls, while stress-free for the upper surface was imposed. All boundaries of the flow domain are thermally insulated. The 3D simulations were conducted using uniform grids in the azimuthal direction and non-uniform grids in the radial direction and axial direction with clustering at the walls (at least 10 grid points were placed inside both the bottom Ekman and sidewall boundary layers). For the run, $96 \times 351 \times 151$ grid points were used in the azimuthal radial and axial directions respectively. The details of the numerical method and validation tests are described in Verzicco and Camussi (1997), Verzicco and Camussi (1999) and Smirnov et al. (2009).

Previous laboratory measurements with salt stratification (Kanda 2004) showed the formation of columnar vortices in experiments, and others (Greenspan 1980; Linden and Van Heijst 1984; Hewitt et al. 1999; Flór et al. 2004) the corner regions and baroclinic instability. To the best of our knowledge, this is the first numerical study in which observations of columnar vortices have been obtained in an spin-up flow with thermal stratification. Our simulations revealed that azimuthal asymmetry is manifested in the form of cyclonic and anticyclonic columnar eddies, which develop at the temperature front formed by the highly distorted isotherms near the cylinder sidewalls (see Figs. 1, 2, 3). Strong deformation of the initial temperature field is caused by the Ekman transport near the bottom. The front steepens until it reaches a quasi-equilibrium state. The eddies grow in size and march along the circumference until they occupy a large portion of the tank. Our goal is to identify and observe the vortex structures generated during the spin-up of the flow. We employ the Q-criterion (Hunt et al. 1988) to identify the vortex, which is defined as a region where the rate of rotation $\Omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i})$ is greater than the rate of strain $S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$.

We use streamlines artificially thickened for visualization purposes that we call velocity tubes. Figures 1, 2 and 3 show the fluid temperature (top), velocity tubes (middle) and vortex structure Q (0) colored by the fluid temperature (bottom) at three different times during the evolution of the spin-up current. The morphology of the columnar vortices is similar to a deformed oblate hemispheroid, i.e. open at the top of the cylindrical container due to the free-slip boundary condition but closed at the bottom.

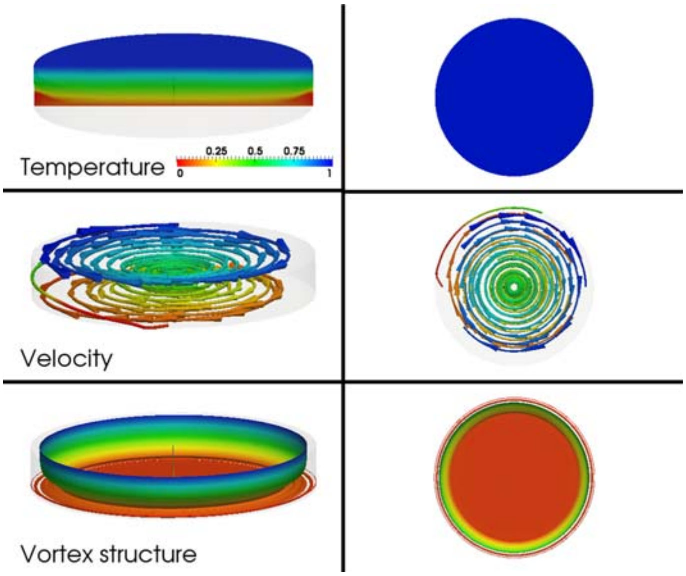

Fig. 1 Temperature field, velocity tubes and vortex structured identified by the isosurfaces of $Q = 0$ colored by temperature at $t = 1$. The *top* views are shown on the *right*

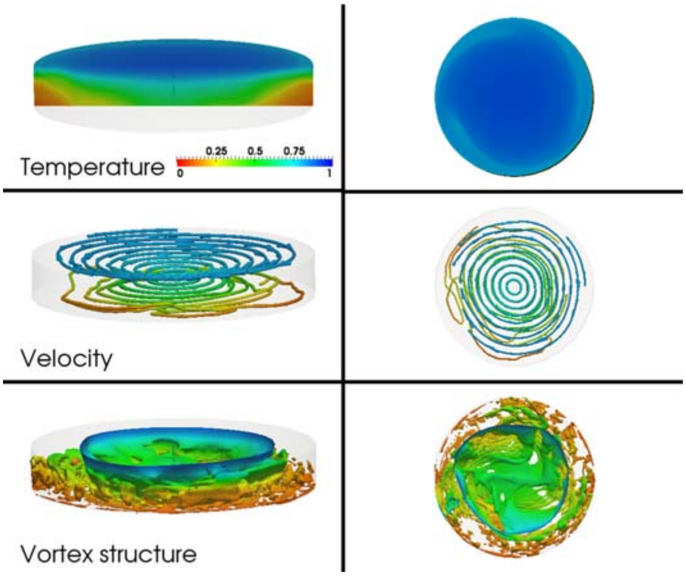

Fig. 2 Temperature field, velocity tubes and vortex structured identified by the isosurfaces of $Q = 0$ colored by temperature at $t = 20$. The *top* views are shown on the *right*

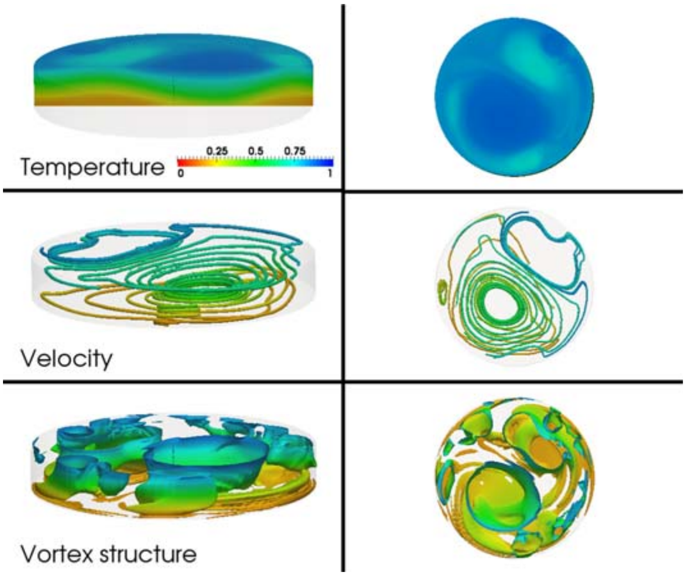

Fig. 3 Temperature field, velocity tubes and vortex structured identified by the isosurfaces of $Q = 0$ colored by temperature at $t = 40$. The *top* views are shown on the *right*

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